Adam Mickiewicz University
Faculty of Mathematics and Computer Science

Geometry and Topology Seminar

1:45 pm, Wednesday, October 17, 2018
B2-38, Collegium Mathematicum

Speaker: Siniša Vrećica (Mathematical Institute, SASA, Belgrade)

Title: Chessboard complexes

Abstract:

A crucial and one of the most significant properties of Algebraic topology is the applicability of its results in different areas of Mathematics. Many important results are established in this way, starting with The fundamental theorem of algebra, Brouwer fixed point theorem, the ham sandwich theorem. In some cases a quite unexpected application of topological result provided a solution to the long-standing conjecture (such as L. Lovász proof of Kneser conjecture). The principal example of such topological result is famous Borsuk-Ulam theorem.

We illustrate the applicability of topological methods and results by presenting an important configuration space - chessboard complex, and by showing how its properties could be used in solving the problems in other areas of Mathematics. Chessboard complex appears in different ways: as a coset complex of the symmetric group by some of its subgroups (stabilizing some elements), as a matching complex of a complete bipartite graph, as a complex of partial injective functions from one finite set to the other, as a deleted join of a finite set (or a repeated deleted join of a point).

Actually we define several versions of this complex and show how each of them is motivated by some mathematical question. For example, we show how a cycle-free chessboard complex appears in establishing the symmetric analogue of the cyclic homology of algebras, and how generalized and symmetrized versions appear in establishing the generalizations of van Kampen-Flores theorem and Tverberg-type theorems.

Our dominant interest is in the connectivity properties of a chessboard complex (which reduces to determining its homology groups), but we consider some other properties as well.

The talk is based on joint papers with R. Živaljević, and some with D. Jojić.

References

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