Inverse Limits of Burnside Rings for p-Groups

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Abstract

Let G be a finite group and H a subgroup of G. Also let V be an $\mathbb{R}[G]$-module and we denote by $V\bullet$ the one point compactification of V. Moreover, let $\mathcal{F}$ be a set of subgroups of G and $\{f_H\}_{H\in\mathcal{F}}$ a family of H-map $f_H : V^\bullet \to V^\bullet$. Then we have a problem that does there exist a G-map $f_G : V^\bullet \to V^\bullet$ such that $f_G \sim_{H-\mathrm{ht}} f_H (\forall H \leq G)$? We denote by $A(G)$ the Burnside ring of G. By evaluating $\mathrm{Coker}(\mathrm{res}^G_{\mathcal{F}} : A(G) \to \prod_{H\in\mathcal{F}} A(H))$, we can know the difficulty of above problem. We denote by $L(G,\mathcal{F})$ the inverse limit of $A(G)$ and by $B(G,\mathcal{F})$ the image of $\mathrm{res}^G_{\mathcal{F}}$, then it is known that $\mathrm{Coker}(\mathrm{res}^G_{\mathcal{F}})$ can be decomposed into the direct sum of $L(G,\mathcal{F})/B(G,\mathcal{F})$ and $P(G,\mathcal{F})/B(G,\mathcal{F})$. Moreover we define $Q(G,\mathcal{F}) = L(G,\mathcal{F})/B(G,\mathcal{F})$, Morimoto showed that $Q(G,\mathcal{F})$ is isomorphic to the product of $Q(G/G^p, \mathcal{F}_{G/G^p})$ for p which devides k_G, where k_G is Oliver's number and G^p is the smallest normal subgroup of G with $|G/G^p|$ p-power. Because G/G^p is p-group, it is important to calculate $Q(G,\mathcal{F})$ in case G is p-group. Hara and Morimoto calculated $Q(G,\mathcal{F})$ in the case of $G = A_4$ the alternating group of degree 4. They also calculated $Q(G,\mathcal{F})$ in case $G = C_p, C_p \times C_p, C_{p^n}$ and $C_p \times C_q$ as p-group, where p and q are distinct primes and n is a positive integer. However, little is known $Q(G,\mathcal{F})$ in the case of G more complicated. Let p be a prime and m and n positive integers. In this talk, we'll calculate $Q(G,\mathcal{F})$ in the case of $G = C_{p^m} \times C_{p^n}$ as application, where C_{p^m} and C_{p^n} are the cyclic groups of order p^m and p^n respectively. We denote by $\mathcal{S}(G)$ the set of all subgroups of G. We show mainly the following two results.

Theorem 1. $\mathcal{F} = \mathcal{S}(G) \setminus \{G\}$. If $G = C_{p^m} \times C_p$, then $Q(G,\mathcal{F}) \cong \mathbb{Z}_p^{\oplus p(m-1)+1}$.

Theorem 2. $\mathcal{F} = \mathcal{S}(G) \setminus \{G\}$. If $G = C_{p^m} \times C_{p^2}$, then $Q(G,\mathcal{F}) \cong \mathbb{Z}_p^{\oplus (p^2+1)(m-2)+p^2+p+2}$.