Adam Mickiewicz University
Faculty of Mathematics and Computer Science

Geometry and Topology Seminar

10:15 am, Tuesday, November 28, 2017
B1-38, Collegium Mathematicum

Speaker: Łukasz Michalak (Adam Mickiewicz University)

Title: The Reeb graph of a function on a manifold with finitely many critical points

Abstract:

The Reeb graph $\mathcal{R}(f)$ of a function f on a manifold M is the quotient space obtained by contracting the connected components of the level sets of function f to the points. For closed manifold and function with finitely many critical points this space is homeomorphic to finite graph, i.e. to one-dimensional finite CW-complex. In this talk we discuss the following problem: which graph can be realized as the Reeb graph of a function on given manifold. The main tool we use is Morse theory and handle decomposition of a manifold.

First we consider the number of cycles in the Reeb graph. We investigate the maximal number of cycles $\mathcal{R}(M)$ in Reeb graph of a function on a manifold M with finitely many critical points and we show that it is maximized by simple Morse functions. We also prove that for every number k no greater than $\mathcal{R}(M)$ there exists Morse function f on manifold M which Reeb graph has exactly k cycles. It turns out that $\mathcal{R}(M)$ is equal to corank of fundamental group of manifold M, i.e. to the maximal rank of free group onto which there exists epimorphism from fundamental group of M.

We also give generalized version of Sharko and Masumoto-Saeki theorem: for any good oriented finite graph Γ there exist some manifold M of a given dimension and Morse function f on M such that the Reeb graph of function f is isomorphic to Γ. After that we can answer the initial question in the case of surfaces.