Speaker:  Gregory Lupton (Cleveland)

Title:  Homotopy Theory in the Digital Topology Setting

Abstract:

I will present a progress report on some work joint with John Oprea and Nick Scoville. An n-dimensional digital image is a finite subset of the integer lattice in $\mathbb{R}^n$, together with an adjacency relation. For instance, a 2-dimensional digital image is an abstraction of an actual digital image consisting of pixels. Our work consists of developing notions and techniques from homotopy theory in the setting of digital images.

In an extensive literature, a number of authors have introduced concepts from topology into the study of digital images. But some of these notions, as they appear in the literature, do not seem satisfactory from a homotopy point of view. Indeed, some of the constructs most useful in homotopy theory, such as cofibrations and path spaces, are absent from the literature. Working in the digital setting, we develop some basic ideas of homotopy theory, including cofibrations and path fibrations, in a way that seems more suited to homotopy theory. We illustrate how our approach may be used, for example, to study Lusternik-Schnirelmann category and topological complexity in a digital setting. One future goal is to develop a characterization of a "homotopy circle" (in the digital setting) using the notion of topological complexity. This is with a view towards recognizing circles, and perhaps other features, using these ideas. This talk will include a survey of the basics on topological notions in the setting of digital images.